Lesson 1: Introduction to Functions and Graphing

- **Goal**: Understand functions, their notation, and how they can be represented visually.
- **Activity**:
    - Mentor explains what a function is using simple examples like f(x)=2x+1f(x) = 2x + 1f(x)=2x+1.
    - Learner plots several points of the function on graph paper or a calculator.
    - Mentor demonstrates how to interpret a graph to understand the behavior of the function (increasing, decreasing, linear, non-linear).
    - Learner practices plotting and interpreting different functions.

Lesson 2: Limits and Intuition of Calculus

- **Goal**: Introduce the concept of limits and how they lead to understanding calculus.
- **Activity**:
    - Mentor shows how to evaluate limits graphically and numerically (e.g., limx→2(x2−1)\lim_{x \to 2} (x^2 - 1)limx→2(x2−1)).
    - Learner tries several examples, working through the steps of approaching a limit from both sides of the graph.
    - Mentor and learner discuss what it means for a limit to not exist or approach infinity.

Lesson 3: Limits at Infinity and Asymptotes

- **Goal**: Understand limits as x→∞x \to \inftyx→∞ and the concept of asymptotes.
- **Activity**:
    - Mentor presents the idea of horizontal and vertical asymptotes using a rational function like f(x)=1xf(x) = \frac{1}{x}f(x)=x1.
    - Learner practices finding limits at infinity and identifying horizontal asymptotes.
    - Mentor guides the learner through the behavior of functions at infinity and how it translates to real-world scenarios.

Lesson 4: Continuity and Discontinuities

- **Goal**: Explore what makes a function continuous and the types of discontinuities.
- **Activity**:
    - Mentor introduces the formal definition of continuity and shows examples of continuous and discontinuous functions.

- Learner identifies points of discontinuity (e.g., holes, jumps, asymptotes) in a graph.
- Mentor gives feedback as the learner works through real examples.

Lesson 5: Introduction to Derivatives (Concept of Rate of Change)

- **Goal**: Introduce derivatives as the rate of change or slope of a function at a point.
- **Activity**:
  - Mentor explains the concept of the derivative with a simple linear function and its slope.
  - Learner practices finding the slope of different linear and non-linear functions at various points.
  - Mentor shows how this leads to the idea of instantaneous rate of change and the derivative formula.

Lesson 6: Basic Derivatives Using Power Rule

- **Goal**: Learn to differentiate basic polynomial functions using the power rule.
- **Activity**:
  - Mentor introduces the power rule for differentiation f(x)=xn→f′(x)=nxn−1f(x) = x^n \rightarrow f'(x) = nx^{n-1}f(x)=xn→f′(x)=nxn−1.
  - Learner practices applying the rule to different functions like x3,x4,2x5x^3, x^4, 2x^5x3,x4,2x5.
  - Mentor guides learner through several examples to reinforce the rule.

Lesson 7: Derivatives of Basic Trigonometric Functions

- **Goal**: Understand and apply derivatives of sine and cosine.
- **Activity**:
  - Mentor explains how to differentiate sin(x)\sin(x)sin(x) and cos(x)\cos(x)cos(x).
  - Learner practices finding the derivatives of these and related functions.
  - Mentor provides real-world applications (e.g., periodic motion, waves) to give context.

Lesson 8: Product and Quotient Rules

- **Goal**: Use the product and quotient rules to differentiate more complex functions.

- **Activity**:
    - Mentor introduces the product rule and quotient rule formulas.
    - Learner practices differentiating products and quotients of functions using the rules.
    - Mentor assists with step-by-step breakdowns, ensuring understanding.

Lesson 9: Chain Rule

- **Goal**: Apply the chain rule to differentiate composite functions.
- **Activity**:
    - Mentor demonstrates the chain rule and explains how it works using functions like f(g(x))f(g(x))f(g(x)).
    - Learner practices differentiating composite functions step by step.
    - Mentor offers feedback, helping the learner navigate through more complex examples.

Lesson 10: Applications of Derivatives (Tangent Lines and Optimization)

- **Goal**: Apply derivative concepts to find tangent lines and solve optimization problems.
- **Activity**:
    - Mentor guides learner in finding the equation of a tangent line at a given point using the derivative.
    - Learner solves basic optimization problems (e.g., maximizing area, minimizing cost).
    - Mentor provides real-world scenarios to apply the optimization techniques.

Teaching Instructions

Lesson 1: Introduction to Functions and Graphing

1. Explain what a function is using a basic example (e.g., f(x)=2x+1f(x) = 2x + 1f(x)=2x+1).
2. Show how to plot points on a graph by picking values for xxx and calculating f(x)f(x)f(x).
3. Guide the learner as they plot their own points and draw the graph.
4. Discuss how the graph shows whether the function is increasing or decreasing.
5. Let the learner plot and interpret new functions, offering feedback.

Lesson 2: Limits and Intuition of Calculus

1. Show what a limit looks like on a graph using a simple example (e.g., limx→2(x2−1)\lim_{x \to 2} (x^2 - 1)limx→2(x2−1)).
2. Demonstrate how to approach a limit from both sides of the graph.
3. Guide the learner as they evaluate limits numerically and graphically.
4. Explain how limits can “not exist” or approach infinity.
5. Let the learner work through several examples, correcting them as needed.

Lesson 3: Limits at Infinity and Asymptotes

1. Explain what limits at infinity are using a rational function (e.g., f(x)=1xf(x) = \frac{1}{x}f(x)=x1).
2. Show how to find the limit as x→∞x \to \inftyx→∞ and identify any horizontal asymptotes.
3. Help the learner practice finding limits at infinity.
4. Guide them through interpreting the behavior of the graph at large values of xxx.
5. Review the concept of asymptotes and their significance.

Lesson 4: Continuity and Discontinuities

1. Explain the formal definition of continuity using an example function.
2. Show what a continuous function looks like on a graph and what a discontinuous one looks like.
3. Guide the learner as they identify points of discontinuity (e.g., holes, jumps).
4. Let the learner find and explain different discontinuities in given functions.
5. Provide feedback on their understanding of continuity and how it applies to real-world problems.

Lesson 5: Introduction to Derivatives (Rate of Change)

1. Explain how a derivative represents the rate of change or slope at a point on a graph.
2. Use a simple linear function to show how the slope is calculated.
3. Guide the learner in calculating the slope at different points for various functions.
4. Discuss how the concept of slope extends to non-linear functions.
5. Let the learner practice finding rates of change, offering corrections as necessary.

Lesson 6: Basic Derivatives Using Power Rule

1. Introduce the power rule f(x)=xn→f′(x)=nxn−1f(x) = x^n \rightarrow f'(x) = nx^{n-1}f(x)=xn→f′(x)=nxn−1.
2. Demonstrate how to differentiate a simple function using the power rule.
3. Let the learner practice on functions like x3,x4,2x5x^3, x^4, 2x^5x3,x4,2x5, etc.
4. Guide them step-by-step, correcting mistakes and reinforcing the rule.
5. Help them understand how this rule simplifies finding derivatives of polynomial functions.

Lesson 7: Derivatives of Basic Trigonometric Functions

1. Explain how to differentiate sine and cosine functions.
2. Show the derivative of sin(x)\sin(x)sin(x) and cos(x)\cos(x)cos(x), explaining their patterns.
3. Guide the learner as they differentiate basic trigonometric functions.
4. Provide real-world examples (like waves or periodic motion) to contextualize the derivatives.
5. Let the learner practice, offering guidance and corrections.

Lesson 8: Product and Quotient Rules

1. Introduce the product rule: (fg)′=f′g+fg′(fg)' = f'g + fg'(fg)′=f′g+fg′.
2. Demonstrate how to apply the product rule with a step-by-step example.
3. Guide the learner as they practice differentiating products of functions.
4. Introduce the quotient rule: (f/g)′=f′g−fg′g2(f/g)' = \frac{f'g - fg'}{g^2}(f/g)′=g2f′g−fg′ and walk through an example.
5. Let the learner practice on functions, offering feedback and corrections as they go.

Lesson 9: Chain Rule

1. Explain the chain rule using a composite function like f(g(x))f(g(x))f(g(x)).
2. Demonstrate how to differentiate composite functions with a simple example.
3. Let the learner practice applying the chain rule to new functions.
4. Correct their mistakes and ensure they understand the steps involved in using the chain rule.
5. Provide progressively more challenging examples to deepen their understanding.

Lesson 10: Applications of Derivatives (Tangent Lines and Optimization)

1. Explain how derivatives can be used to find the slope of a tangent line at a point.
2. Walk the learner through an example of finding the tangent line equation for a function.
3. Introduce optimization problems, such as maximizing area or minimizing cost.
4. Let the learner practice solving basic optimization problems with your guidance.
5. Offer feedback and help them apply the derivative concepts to real-world scenarios.